Undergraduate Research in Mathematics and Statistics

Rowan's Department of Mathematics
3/2/2022

What is student research?

- Collaborate with a mentor (professor or industrial supervisor) on a research project to solve a mathematical/statistical problem of interest to the academic community or motivated by industry.
- Devote a certain number of hours/week to work on the project and meet regularly with their mentor.
- Different types of problems: theoretical, applied (industry), computational (programming)
- Different opportunities: internal/external to Rowan, paid/unpaid, internships, REUs.

Why do research as an Undergrad?

- Educational Benefits
- Professional Benefits
- Personal Benefits

Educational Benefits

- Enhance understanding and knowledge of your academic field.
- Apply what you learn in class to real-world projects.
- Learn new things that aren't covered in classes
- Earn academic credit
- Prepare for graduate study
- Improve/Acquire Skills:
  - communication (written and oral)
  - critical thinking & problem-solving
  - teamwork
  - time management

Professional Benefits

- Explore your interests and clarify your career goals
- Strengthen your resume
- Develop strong relationships with faculty (think recommendation letters!!!).
- Network with experts in your field (potential future employers).
- Check out potential graduate school programs (off campus)

Personal Benefits

- Build confidence in your skills.
- Sharpen your critical and analytical thinking skills
- Travel to a new place (off campus research or conference presentation)
- Earn scholarships, stipends, and/or awards
- …and more

Where can you find research opportunities?

- On Campus:
  - Work with our faculty members
  - Paid or Unpaid options
  - Semester long or Summer (5 or 10 weeks available)
- Off campus
  - Usually over summer
  - Search internet (Key words: Summer undergrad research/internships)
  - Check out Local Businesses (Large companies: Banks, Pharmaceuticals, etc.)
  - National Science Foundation REU (Research Experience for Undergraduates) program: https://www.nsf.gov/crssprgm/reu/

How do you start?

- Prepare yourself:
  - Think about the kind of research you want to do. Be open.
  - Strengthen your skill sets (consider taking Math 01.390 – Mathematical Research)
  - Plan early (deadlines may be in Dec-April)

- Talk to your professors (and other faculty):
  - To learn about different research topics
  - Ask about research opportunities with them
  - Ask for recommendation letters (for off campus opportunities)

Math 01.390 – Mathematical Research

- Offered in fall semesters. To enroll, contact Dr. Thanh Nguyen (nguyent@rowan.edu)
- Provides appropriate research problems for you to do in one semester (and beyond if you want)
- Connects you with Math research faculty
- Strengthen:
  - Research skills (analytical, statistical, computational)
  - Writing skills (latex for math writing)
  - Presentation skills (present your research project)

Preparing applications for off-campus opportunities

- Resume:
  - Write a good one (get help)
  - List of relevant coursework or transcripts

- Letter(s) of Recommendation
  - Don't wait until the last minute

- Personal Statement (important)
- Ask for a second opinion from advisor

Math Faculty Research Interests & potential research projects

Dr. Nasrine Bendjilali

- Research Interests:
  - Machine learning, neural networks and analysis of big data.
  - Genetic risk factors contributing to development of complex human diseases
  - Statistical methods for genetic mapping of human traits
  - Multiple hypothesis testing procedures and their applications in biomedical sciences
- Potential Projects:
  - Machine learning, neural networks and analysis of big data.
- Potential for summer and future funding.
- Time frame: Starting Spring 2022 semester
- Student Skills required:
  - Motivation to do research
  - Background in programming using Python or R.
- Contact Info:

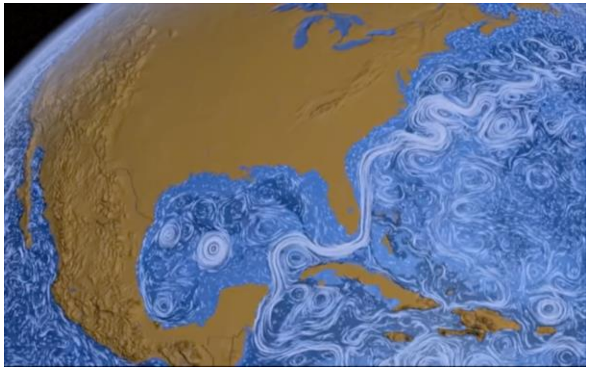
bendjilali@rowan.edu , office: Robison Hall 229 C

Dr. Abdul Hassen

- Research Interests:
  - Analytic Number Theory
  - Partition Functions
  - Bernoulli and Euler Polynomials and Numbers
- Potential Projects:
  - Generalized Euler Numbers
  - Convolutions Properties of Generalized Euler Polynomials
  - Zeros of Generalized Euler Polynomials
- Looking for two committed students
- Time frame: Late Spring 2022 and beyond!
- Student Skills required:
  - Discrete math, Calculus III
- Contact Info:
  - hassen@rowan.edu, Robinson 229E

Dr. Helga Huntley

- Research Interests:
  - Geophysical fluid dynamics.
  - Ocean, atmosphere, climate modeling.
  - State estimation and predictability.
- Potential Projects:
  - Extract flow properties from observed trajectories of drifters.
  - Large vortex spin-off in the Gulf of Mexico.
  - Image analysis of photos of floating objects.
- Time frame: Summer 2022 and thereafter
- Funding possible
- Student Skills required:
  - Curiosity about the ocean.
  - Programming skills in Matlab.
  - Multivariable calculus; differential equations a plus. (Precalculus only for image analysis)
- Contact Info:
  - huntleyh@rowan.edu.



Dr. Ik Jae Lee

- Research Interests:
  - Knot Theory, Low Dimensional Topology
- Potential Projects:
  - Understanding triangulations on 3 or 4 dimensional manifolds with Regina (a topology software)
- Time frame: Starting fall 2022 / spring 2023 semester
- Student Skills required:
  - Computer programming skills
  - Discrete math
- Contact Info:
  - leei@rowan.edu, Robinson 228J

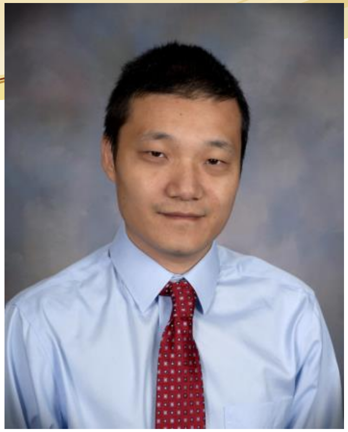
Dr. Hieu Nguyen

- Research Interests:
  - Coding Theory, Deep Learning (AI)
- Potential Projects:
  - Error-correcting codes and decoding algorithms with applications to DNA barcoding
  - AI in agriculture: Blueberry fruit detection using autonomous drones
  - AI in medicine: Aneurysm detection through CT imaging
- Looking for up to 3 students
- Funding: potentially available for Fall 2022 to support 2 students
- Time frame: Start ASAP on volunteer basis (until funding is available)
- Student Skills required:
  - Discrete math
  - Strong computer programming skills
- Contact Info:
  - nguyen@rowan.edu, Robinson 228N

Dr. Thanh Nguyen

- Research Interests:
  - Differential equations; Numerical analysis; Optimization; Scientific machine learning; Applications in engineering and industry
- Projects for 2022-2023:
  - Model & source identification in water pollution modeling
  - Modeling Covid-19 spread using differential equations & machine learning
- Looking for at least 4 students
- Funding:
  - may be 1 stipend in summer 2022
  - At least 4 stipends available for Sep 2022 – May 2023.
- Time frame: Start ASAP
- Student Skills required:
  - Linear algebra; differential equations (good but not necessary)
  - Programming skills (Python/Matlab)
- Contact Info: nguyent@rowan.edu, Robinson 230C

Dr. Juming Pan



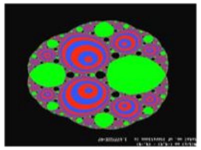
- Research Interests:
  - Causal Inference
  - High-dimensional Data Analysis
- Student Skills required:
  - Diligent and enthusiastic about research
  - Easy to communicate with
  - Basic statistics knowledge
  - Some experience in programming such as R, Python.
- Contact Info:
  - pan@rowan.edu, Robinson 230 F.

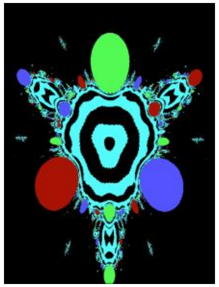
Dr. Uma Thayasivam

- Research Interests:
  - Statistical Learning, Predictive Modeling, Data Science Interpretable AI & Quantum Computing in Machine Learning
- Potential Projects:
  - Student predictive modeling
  - Telemedicine and Telehealth Statistical learning
  - Interpretable AI for Data Science
  - Robust methods for high dimensional data
  - Open to student driven projects on statistical/data mining
- Looking for interesting students who are willing to gain experience
- Initially unfunded with potential for future funding support including summer funding.
- Time frame: Summer 2022 and beyond!
- Student Skills required:
  - Basic Statistics course
  - Some experience in computing programming
  - Desire to learn and willing to commit some quality time
- Contact Info:
  - thayasivam@rowan.edu, Robinson 228 H

Dr. Marcus Wright

- Research Interests (more details available upon request):
  - Geometry of Julia Sets Associated to Root-Finding Methods (Newton, Halley, Secant, ???) (see images below)
  - Constructive Algorithms for Elementary Number Theory (Fermat Two Squares, Factorization, Chirality of Triples/Quadruples)
  - I am willing to collaborate with other professors and their students
- Potential Projects:
  - Using Mathematica to Image/Analyze Julia Sets
  - Understand the Zagier Algorithms for Solutions of the Two Squares Problem
  - Using a constructive algorithm for Zagier's Fermat Two-square proof to factor
  - Generalizing Chirality of Triples to n-tuples
  - Open to student driven projects!
- Looking for up to 2 students
- Unfunded for now
- Time frame: Spring 2022 and beyond!
- Student Skills required:
  - Your brain should be open.
  - Basic programming/Mathematica could be useful for some projects
- Contact Info:
  - wright@rowan.edu, Robinson 229D





Sample research projects

Error Correcting Codes

Liam Busch & Dr. Hieu Nguyen
Department of Mathematics, Rowan University

Binary codewords errors &correction

- **Why this project?**
  - Computers "speak" in strings of 1's and 0's called binary codewords.
  - Sometimes the codewords become corrupted.
    - A bit could be deleted: 10110 → 1110
    - Or inserted: 10110 → 101**0**10
    - Or replaced: 10110 → 10**0**10
- **Our research goal**: find algorithms to DETECT and CORRECT possible errors in codewords

Our developed algorithm

- We have discovered a new algorithm for correcting these corruptions.
  - We found this algorithm through generating data and looking for patterns
  - We used codes that we programmed ourselves
  - We updated the algorithm until every codeword it generated was recoverable
- The algorithm uses linear algebra, number theory, and modular arithmetic.

A Demonstration

- Can you make a codeword that can't be decoded?
- Up to two errors are allowed.
  - This means that you can do any of the following:
    - Insert a bit
    - Delete a bit
    - Insert two bits
    - Delete two bits
    - Insert a bit and delete a bit

Hypergeometric Zeta Function of Order Two

Courtland Karpolorich & Dr. Abdul Hassen
Department of Mathematics, Rowan University

Classical zeta function

- Euler's Discoveries

$$\zeta(2) = \frac{\pi^2}{6}$$

$$\zeta(2n) = \frac{(-1)^{n+1} B_{2n} (2\pi)^{2n}}{2(2n)!}$$

$$\zeta(s) = \prod_p \frac{1}{1 - p^{-s}}$$

$$\sum_p \frac{1}{p} \sim \log(\sum_n \frac{1}{n})$$

- Riemann's Discoveries
  - Analytic Continuation
  - Riemann Hypothesis

$$\zeta(s) = 2(2\pi)^{s-1} \sin\left(\frac{\pi}{2} s\right) \Gamma(1-s) \zeta(1-s)$$

The Zeta Function has uses in physics, probability theory, statistics, and cryptography. The more we find out, the more we can research and expand our knowledge.

Goal of project

- Study the properties of the now generalized Zeta Function.

$$\zeta_N(s) = \frac{1}{\Gamma(s+N-1)} \int_0^\infty \frac{x^{s+N-2}}{e^x - T_{N-1}(x)} dx$$

- We look particularly at when N=2.

$$\zeta_2(s) = \frac{1}{\Gamma(s-1)} \int_0^\infty \frac{x^s}{e^x - 1 - x} dx.$$

$$\zeta_2(s) = \sum_{n=0}^{\infty} \frac{\mu_2(n,s)}{n^{s+1}}.$$

Results

- Second Order Bernoulli Numbers

$$\frac{x^2/2}{e^x-1-x}=\sum_{n=0}^{\infty}\frac{B_{2,n}}{n!}x^n$$

$$\begin{aligned} B_{2,0} &= 1, \\ B_{2,1} &= -\frac{1}{3}, \\ B_{2,m} &= -\sum_{k=0}^{m-1}\binom{m}{k-2}\frac{2B_{2,k}}{k(k-1)}+\frac{2(m-1)}{3(m+1)(m+2)}, \quad m\geq 2. \end{aligned}$$

- Dirichlet Series Representation

$$\zeta_2(s)=\sum_{n=1}^{\infty}\sum_{k=0}^{n-1}\frac{c_k(n)s^k}{n^{s+1}}.$$

$$\sum_{n=0}^{\infty}\frac{(-1)^{k+1}}{(k+1)!}(log(1+W(-n)))^{k+1}$$

Causal Inference: Propensity Score Matching

Dr. Juming Pan

What is causal inference?

Inferring the effects of any treatment/policy/intervention/etc.

Examples:

- Effect of treatment on a disease
- Effect of climate change policy on emissions
- Effect of social media on mental health
- Many more (effect of X on Y)

What are propensity scores for?

You want to know the effect of something

- You don't have random assignment
- You do have data on pre-program characteristics that determined whether or not the individual received the treatment

An example

- An NGO has built health clinics in several villages.
- Villages were not selected randomly
- You have data on village characteristics before the program was implemented
- What's the effect of the program on infant mortality?

	T	imrate
1.	1	10
2.	1	15
3.	1	22
4.	1	19
5.	0	25
6.	0	19
7.	0	4
8.	0	8
9.	0	6

An example

- An NGO has built health clinics in several villages.
- Villages were not selected randomly
- You have data on village characteristics before the program was implemented
- What's the effect of the program on infant mortality?

	T	imrate
1.	1	10
2.	1	15
3.	1	22
4.	1	19
5.	0	25
6.	0	19
7.	0	4
8.	0	8
9.	0	6
10.		

$$0.25(10+15+22+19) - 0.2(25+19+4+8+6) = +4.1$$

Looks like clinics are **increasing** infant mortality!

An example

- An NGO has built health clinics in several villages.
- Villages were not selected randomly
- You have data on village characteristics before the program was implemented
- What's the effect of the program on infant mortality?

	T	imrate	povrate	pcdocs
1.	1	10	.5	.01
2.	1	15	.6	.02
3.	1	22	.7	.01
4.	1	19	.6	.02
5.	0	25	.6	.01
6.	0	19	.5	.02
7.	0	4	.1	.04
8.	0	8	.3	.05
9.	0	6	.2	.04

The basic idea

1. **Create a new control group:**
   For each observation in the treatment group, select the control observation that looks most like it based on the selection variables (aka background characteristics)

2. **Compute the treatment effect:**
   Compare the average outcome in the treatment group with the average outcome in the new control group

Predicting selection

How do you actually match treatment observations to controls?

Use logistic (or probit) regression to estimate

$$Prob(T=1 \mid X_1, X_2, \ldots, X_K)$$

An example

- An NGO has built health clinics in several villages.
- Villages were not selected randomly
- You have data on village characteristics before the program was implemented
- What's the effect of the program on infant mortality?

	T	imrate	povrate	pcdocs	ps1
1.	1	10	.5	.01	.4165713
2.	1	15	.6	.02	.7358171
3.	1	22	.7	.01	.9284516
4.	1	19	.6	.02	.7358171
5.	0	25	.6	.01	.752714
6.	0	19	.5	.02	.395162
7.	0	4	.1	.04	.0016534
8.	0	8	.3	.05	.026803
9.	0	6	.2	.04	.0070107

Predicted prob of treatment

aka the propensity score

An example

- An NGO has built health clinics in several villages.
- Villages were not selected randomly
- You have data on village characteristics before the program was implemented
- What's the effect of the program on infant mortality?

	T	imrate	povrate	pcdocs	ps1	match1
1.	1	10	.5	.01	.4165713	6
2.	1	15	.6	.02	.7358171	5
3.	1	22	.7	.01	.9284516	5
4.	1	19	.6	.02	.7358171	5
5.	0	25	.6	.01	.752714	
6.	0	19	.5	.02	.395162	
7.	0	4	.1	.04	.0016534	
8.	0	8	.3	.05	.026803	
9.	0	6	.2	.04	.0070107	

.25*(10+15+22+19)-0.25*(19+25+25+25) = -7

Model & source identification in water pollution modeling

Dr. Thanh Nguyen

Model & source identification in water pollution modeling

- Supported by the Center for Undergraduate Research (funding from NSF): $3000 for Sep 2022-May 2023.
- Work in a team of 4 students (2 from Rowan, 2 from Camden County College) and 2 faculty (Dr. T Nguyen & Tuan Le, CCC)
- Research training & weekly research seminar
- Opportunities to attend and present at Joint Mathematics Meeting (Boston, MA, Jan 2023) and MathFest (Tampa, FL, July-Aug 2023) (travel expenses covered)

Application: detect pollution sources & modeling of water pollution scenarios

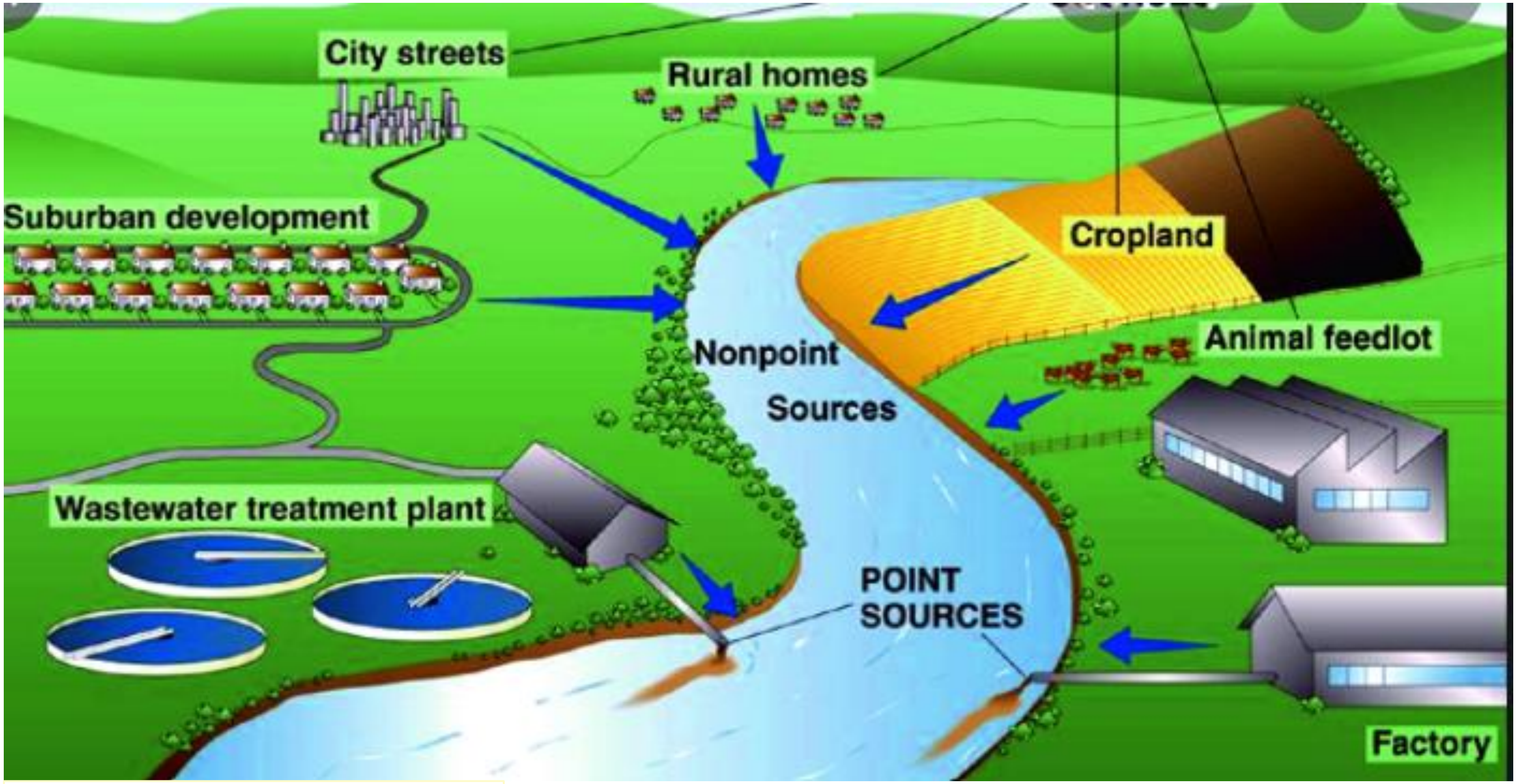


Photo Source: Springer

Mathematical model & goal

- Time-independent model:

$$\frac{dB}{dx} + v\,B(x) = -k_d B(x) + S(x), \qquad 0 < x < L,$$

$$\frac{dD}{dx} + v\,D(x) = k_d B(x) - k_a D(x), \qquad 0 < x < L,$$

  - S(x): pollution source
  - B: Biochemical demand;
  - D: Dissolved oxygen in water (**measurable**)
  - v: velocity of water flow; k_d, ka: model coefficients

- **Goal**: Determine model coefficients k_d, ka and source S(x) from measured data of dissolved oxygen

Mathematical & computational tools used

- Differential equations (ordinary differential equations; partial differential equations)
- Optimization (finding the minimum of a function)
- Numerical methods for solving differential equations
- Data analysis & machine learning
- Programming

Generalized Euler Numbers

Dr. Abdul Hassen

Generalized Euler Numbers

We define the *nth* Euler Numbers of the *Nth* degree, $E_n(N)$, to be generated by the function:

$$G_N(x) = \frac{2e^x}{e^{2x} + T_N(2x)} = \sum_{n=0}^{\infty} E_n(N)\frac{x^n}{n!} \tag{1}$$

where

$$T_N(x) = \sum_{n=0}^{N-1} \frac{(x)^n}{n!} \tag{2}$$

Note that the first degree Euler numbers are the traditional Euler numbers.

$$\frac{2e^x}{e^2x + 1} = \sum_{n=0}^{\infty} E_n\frac{x^n}{n!}.$$

Hyperbolic Euler Numbers and polynomials, defined, respectively, by

$$\frac{1}{\Phi_{N,0}(x)} = \frac{N}{\sum_{n=j}^{N-1} e^{w_N^j x}} = \sum_{n=0}^{\infty} \mathcal{E}_n(N) \frac{x^n}{n!}$$

and

$$\frac{e^{\left(z-\frac{1}{N}\right)x}}{\Phi_{N,0}\left(\frac{x}{N}\right)} = \frac{N e^{\left(z-\frac{1}{N}\right)x}}{\sum_{n=0}^{N-1} e^{\frac{w_N^n x}{N}}} = \sum_{n=0}^{\infty} \mathcal{E}_n(N,z) \frac{x^n}{n!},$$

where

$$\Phi_{N,k}(x) = \frac{1}{N} \sum_{n=0}^{N-1} w_N^{-nk} e^{w_N^n x} = \sum_{n=0}^{\infty} \frac{x^{Nn+k}}{(Nn+k)!}$$

and

$$w_N = e^{\frac{2\pi i}{N}}$$

For $N = 3$, the first 9 nonzero hyperbolic Euler numbers are

$1, -1, 19, -1513, 315523, -136085041, 105261234643, -1327052213993

Generalized Bernoulli Numbers

We define the *nth* Bernoulli Numbers of the *Nth* degree, $B_n(N)$, to be generated by the function:

$$G_N(x) = \frac{1}{e^x - T_{N-1}(x)} = \sum_{n=0}^{\infty} B_n(N) \frac{x^n}{n!} \tag{3}$$

where

$$T_N(x) = \sum_{n=0}^{N-1} \frac{(x)^n}{n!} \tag{4}$$

Note that the first degree Euler numbers are the traditional Euler numbers.

$$\frac{1}{e^x - 1} = \sum_{n=0}^{\infty} B_n \frac{x^n}{n!}.$$

The (N,k)-Hyperbolic Bernoulli Numbers of are defined by the exponential generating function

$$\frac{x^k e^{-\frac{x}{N}}}{N^k k! \Phi_{N,k}\left(\frac{x}{N}\right)} = \sum_{n=0}^{\infty} \mathcal{B}_n(N,k) \frac{x^n}{n!}.$$

The (N,k)-Hyperbolic Bernoulli Polynomials are defined by the exponential generating function

$$\frac{x^k e^{\left(z-\frac{1}{N}\right)x}}{N^k k! \Phi_{N,k}\left(\frac{x}{N}\right)} = \sum_{n=0}^{\infty} \mathcal{B}_n(N,k,z) \frac{x^n}{n!}.$$

Questions?