

Syllabus

Math 01.330 - Introduction to Real Analysis I

CATALOG DESCRIPTION:

Math 01.330 Introduction to Real Analysis I, 3 s.h.

Prerequisites: Math 01.230 (Calculus III) and Math 03.150 (Discrete Math) with a grade of C- or better in both courses

This course introduces students to rigorous mathematical thought processes in Analysis, and prepares them for more advanced courses in the related areas. Topics included are: sets, functions, the real number system, sequences, limits, continuity and derivatives.

OBJECTIVES:

Students will demonstrate the ability to use rigorous mathematical thought processes in the following areas: sets, functions, sequences, limits, continuity, and derivatives.

CONTENTS:

1.0 Introduction

1.1 Real numbers

1.1.1 Absolute values, triangle inequality

1.1.2 Archimedean property, rational numbers are dense

1.2 Sets and functions

1.2.1 Set relations, Cartesian product

1.2.2 One-to-one, onto, and inverse functions

1.3 Cardinality

1.3.1 One-to-one correspondence

1.3.2 Countable and uncountable sets

1.4 Methods of proof

1.4.1 Direct proof

1.4.2 Contrapositive proof

1.4.3 Proof by contradiction

1.4.4 Mathematical induction

2.0 Sequences

2.1 Convergence

2.1.1 Cauchy's epsilon definition of convergence

2.1.2 Uniqueness of limits

2.1.3 Divergence to infinity

- 2.1.4 Convergent sequences are bounded
- 2.2 Limit theorems
 - 2.2.1 Summation/product of sequences
 - 2.2.2 Squeeze theorem
 - 2.2.3 Cauchy sequences and Cauchy criteria for convergence
 - 2.2.4 Completeness axiom
 - 2.2.5 Bounded monotone sequences are convergent
- 2.3 Subsequences and limit points
 - 2.3.1 Bolzano-Weierstrass theorem
- 2.4 Supremum and infimum
 - 2.4.1 A bounded set has a unique least upper bound

3.0 Continuity

- 3.1 Limits of functions
 - 3.1.1 Definition of continuity based on sequences
 - 3.1.2 Definition of continuity based on open intervals
 - 3.1.3 Summation/product/composition of continuous functions
- 3.2 Properties of continuous functions on a closed interval
 - 3.2.1 A continuous function is bounded
 - 3.2.2 A continuous function attains its supremum/infimum
 - 3.2.3 Intermediate-value theorem
 - 3.2.4 Uniform continuity

4.0 Differentiation

- 4.1 Derivatives
 - 4.1.1 Limit definition of a derivative
 - 4.1.2 Rules of differentiation
 - 4.1.3 Chain rule
 - 4.1.4 Higher-order derivatives
- 4.2 Properties of differentiable functions
 - 4.2.1 Differentiability implies continuity
 - 4.2.2 Continuously differentiable functions

TEXT:

*Michael Reed, FUNDAMENTAL IDEAS OF ANALYSIS, WILEY & SON, 1998

Bartle, Robert G. & Sherbert, Donald R., INTRODUCTION TO REAL ANALYSIS, third ed., John Wiley & Sons, Inc., 2000.

Mattuck, Arthur, INTRODUCTION TO ANALYSIS, first ed., Prentice Hall, 1999.